Drude Model for metals

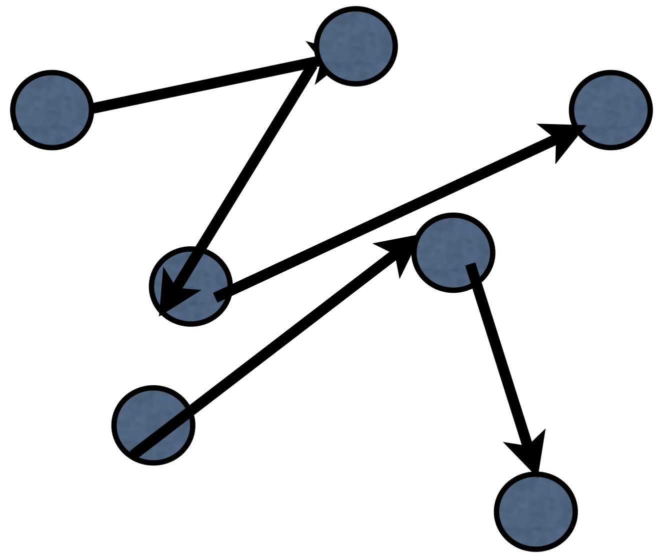
Metals have properties that were not explainable using basic laws of physics:
They have properties that other materials lack (quartz, sulfur, salt...):

- Excellent electric & thermic conductors
- Ductile & malleable
- reflectivity

Modern theory of solids was founded on an attempt to explain all these properties.

Fundamental elements favor metallic state. We need to understand non metallic materials to understand metals: Why gold conducts so well and salt does not?

Drude Model for metals (1900), after Thomson (1897) e^- discovery

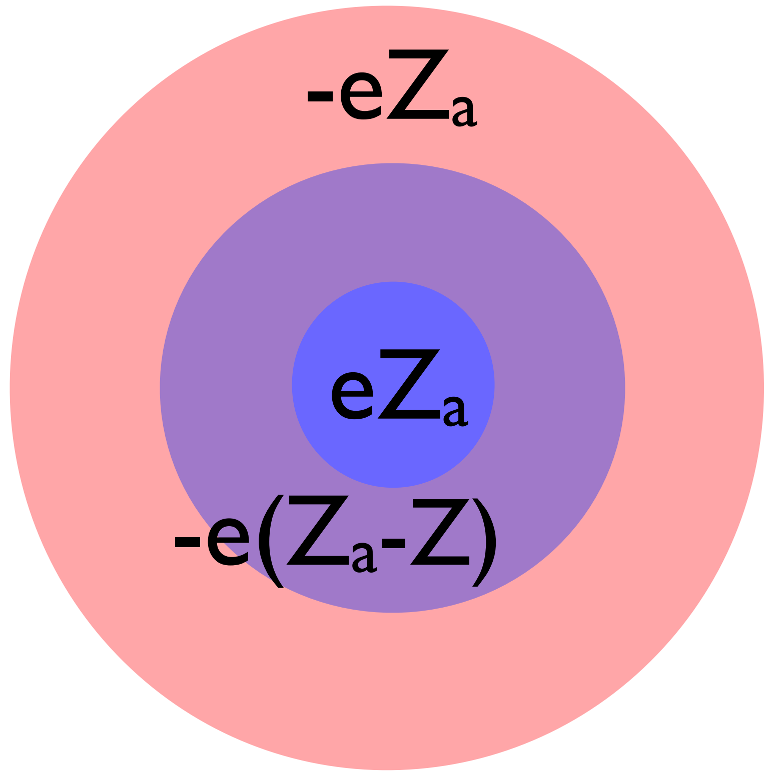


Fundaments: Kinetic theory of gases

- atoms= rigid spheres
- they move in straight lines until they collide

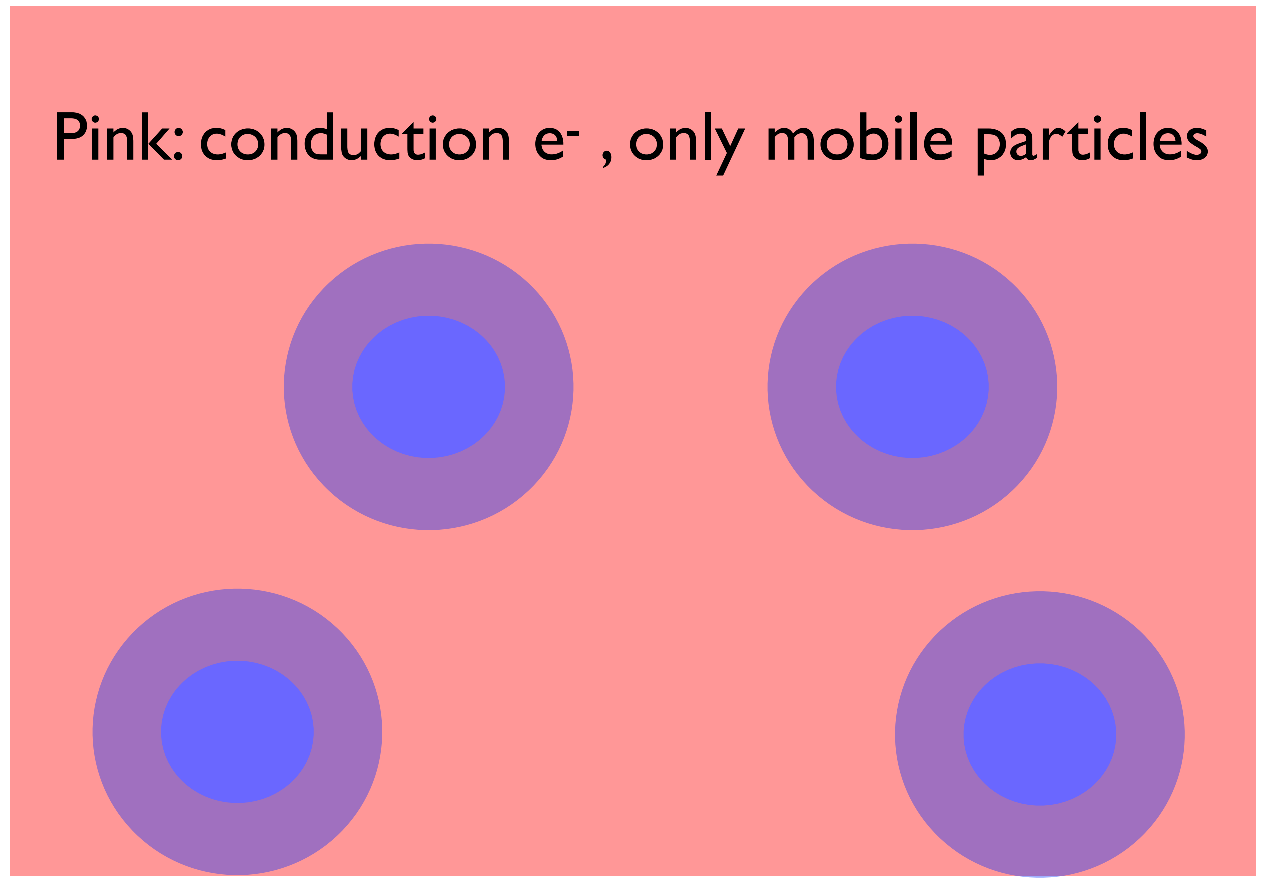
time of collision--> negligible (forces only relevant during collision)

- Metals: Formed by + (heavier, fixed) and - particles.



Pseudopotential:

$$V(\vec{r}) = -eZ/\vec{r}$$



Drude model: Kinetic theory of (conduction) e- of mass m

Some numbers: 1mol N_A--> 6.02 * 10 ^{23} atoms/mol.

$$\rho_m/A \qquad mol/cm^3 \qquad \rho_m = \quad mass\ density$$

$$n = \frac{N \rightarrow \#e^-}{V \rightarrow Volume} = N_A \cdot Z \cdot \frac{\rho_m}{A}$$

Important parameter: r_s

Measurement of the e- density: radius of a sphere with volume (V) = volume per conduction e^-.

$$a_0 = \hbar^2/(me^2) = 0.529\mathring{A} = 1Bohr$$

$$\frac{V}{N} = \frac{1}{n} \Longrightarrow r_s = (\frac{3}{4\pi n})^{1/3}$$

r_s/a_0 is the usual measurement of the density of valence e^- .

large r_s --> sparse, small r_s-->dense

Table 1.1
FREE ELECTRON DENSITIES OF SELECTED METALLIC ELEMENTS[a]

ELEMENT	Z	n ($10^{22}/\text{cm}^3$)	r_s(Å)	r_s/a_0
Li (78 K)	1	4.70	1.72	3.25
Na (5 K)	1	2.65	2.08	3.93
K (5 K)	1	1.40	2.57	4.86
Rb (5 K)	1	1.15	2.75	5.20
Cs (5 K)	1	0.91	2.98	5.62
Cu	1	8.47	1.41	2.67
Ag	1	5.86	1.60	3.02
Au	1	5.90	1.59	3.01
Be	2	24.7	0.99	1.87
Mg	2	8.61	1.41	2.66
Ca	2	4.61	1.73	3.27
Sr	2	3.55	1.89	3.57
Ba	2	3.15	1.96	3.71
Nb	1	5.56	1.63	3.07
Fe	2	17.0	1.12	2.12
Mn (α)	2	16.5	1.13	2.14
Zn	2	13.2	1.22	2.30
Cd	2	9.27	1.37	2.59
Hg (78 K)	2	8.65	1.40	2.65
Al	3	18.1	1.10	2.07
Ga	3	15.4	1.16	2.19
In	3	11.5	1.27	2.41
Tl	3	10.5	1.31	2.48
Sn	4	14.8	1.17	2.22
Pb	4	13.2	1.22	2.30
Bi	5	14.1	1.19	2.25
Sb	5	16.5	1.13	2.14

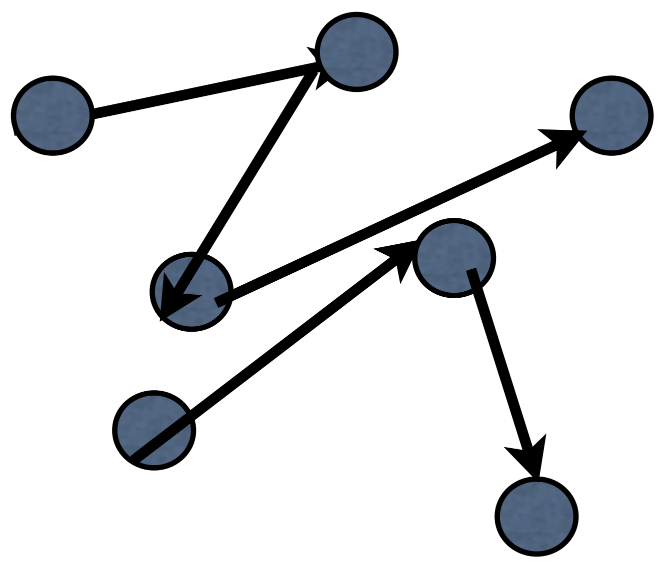
[a] At room temperature (about 300 K) and atmospheric pressure, unless otherwise noted. The radius r_s of the free electron sphere is defined in Eq. (1.2). We have arbitrarily selected one value of Z for those elements that display more than one chemical valence. The Drude model gives no theoretical basis for the choice. Values of n are based on data from R. W. G. Wyckoff, *Crystal Structures*, 2nd ed., Interscience, New York, 1963.

A& M table 1.1

Drude's model assumptions:

1. Electrons move freely (Newton's Laws) between collisions (no e^--e^-, no e^--ion)
2. Abrupt collisions (with ions)
3. Average collision time=τ, ℓ=vτ→ mean free path. $1/\tau \Longrightarrow$ prob. of collision / unit time.
4. Memory washing collisions: v after collisions depends only on the T at the collision site.

$$3/2k_BT = 1/2mv^2$$



Equation of motion for e- (Newton's Law), in the presence of a uniform magnetic or E field.

- Prob. of collision between t and t+dt= dt/τ
- Prob. of no collision =(1-dt/τ)
- e- that do not collide change their momentum by the external field (E): $\vec{f}(t)dt$

$$1 \qquad (1 - dt/\tau)(\vec{p}(t) + \vec{f}(t)dt) = \vec{p}(t) - (dt/\tau)\vec{p}(t) + \vec{f}(t)dt$$

- Contribution to the change in momentum by the e- that have suffered a collision:

$$2 \qquad dt/\tau(\underbrace{\vec{x}}_{0,\ random\ direction} + \vec{f}(t)dt) \approx dt^2 \Longrightarrow neglect$$

- So: $(\vec{p}(t) + dt) = 1 + 2 = 1$

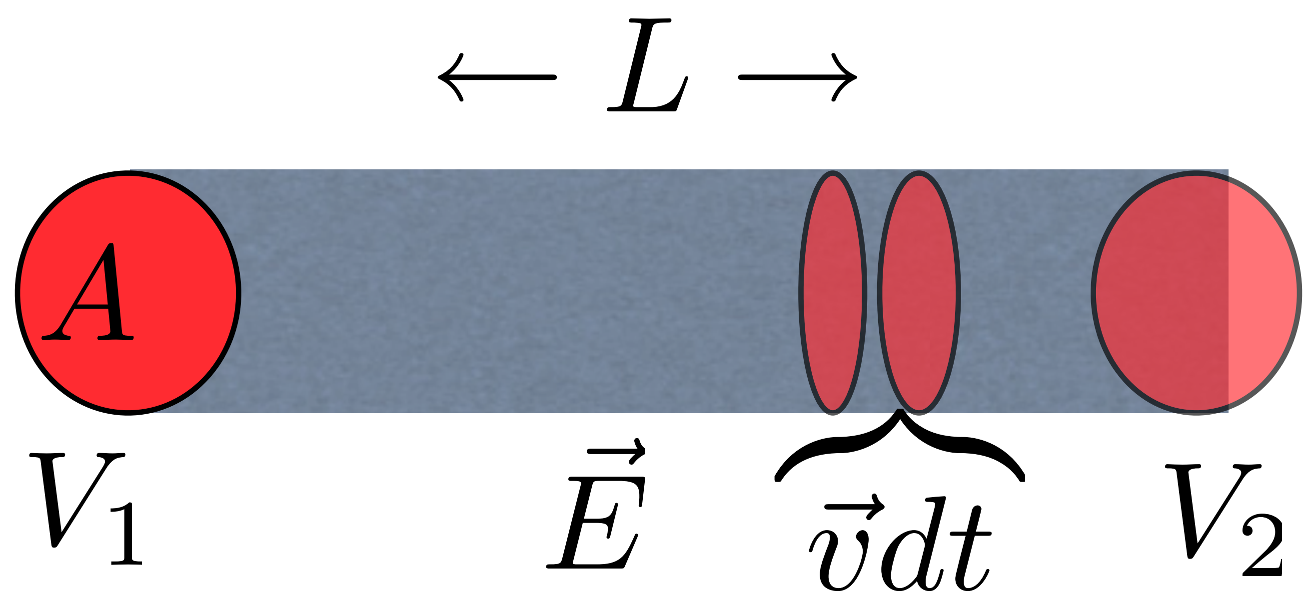
- So, change in momentum of the e- gas : $\frac{\vec{p}(t)}{dt} = \underbrace{-\frac{\vec{p}(t)}{\tau}}_{\text{frictional drag force}} + \vec{f}(t)$

frictional drag force

DC electrical conductivity

- Ohm's Law: V=I R
  - R=Resistance ⇒R depends on wire dimensions but not on V or I.
  - ρ=resistivity ⇒ characteristic of the metal. $\vec{E} = \rho \vec{j}$
  - **j**⇒ current density, // to the flow of charge=$q_{charge}/(t_{ime}\ A_{rea})$=I/A

$$\Delta V = V_2 - V_1 = EL = \rho I/AL \Longrightarrow R = \rho L/A$$



- n e^-/V_{olume} move with velocity $\vec{v}$ current density $\vec{j}//\vec{v}$
- n A |v| dt e^- will cross area A (note q=-e), so in dt the charge crossing the area A =-ne |v| A dt

$$j = q/(tA) = -ne|v|; \vec{j} = -ne\vec{v} \qquad \vec{v} = \vec{p}/m$$

$$DC \rightarrow d\vec{p}/dt = 0 \rightarrow \vec{p}(t) = \tau \vec{f}(t)$$

$$\vec{f} = -e\vec{E} \rightarrow \vec{p} = -e\vec{E}\tau$$

$$\vec{j} = -ne\frac{-e\vec{E}\tau}{m} = \frac{ne^2\tau}{m}\vec{E}$$

$$\vec{j} = \sigma\vec{E} \qquad \leftarrow Ohm's Law$$

$$\sigma = \frac{ne^2\tau}{m} \qquad Drude \quad conductivity$$

- τ can be estimated using observed resistivities. $\tau \sim 10^{-14}, 10^{-15}$ s
- the velocity can be obtained from the classical equipartition of energy

  $1/2\, m v_0^2 = 3/2 k_B T \Rightarrow v_0 \sim 10^7$ cm/s.

$\ell \sim$ 1-10 Å, consistent with Drude's model but not with reality (1 order of magnitude wrong in v_0!) : wrong classical dynamics and wrong picture of scattering.
In reality v_0 is independent of T !!

- τ independent quantities will yield much more realistic information.

$$\sigma_{Drude} = \frac{ne^2\tau}{m}$$

$$\sigma_{Drude,ac} = \frac{ine^2}{m(\omega + i/\tau)}$$

1. These work surprisingly well; ac version is fine.
2. Quantum version is definitely needed. Issues: n/m, 1/τ
3. Bohr's doctoral dissertation clarified problems with 1/τ
4. Sommerfeld's quantum electron gas theory clarified other problems.

example: Quantum theory (Bloch) shows that electrons diffract around atoms, only scattering from impurities; Classical theory says all atoms scatter.

$$\ell\sigma_X = \bar{v}\tau\sigma_X = \frac{1}{n}$$

=volume per atom; σ_X is scattering cross section.

$$\frac{1}{\tau} = n\sigma_X\bar{v} \propto T^{1/2}$$

Classical statistics says resistivity of a metal should scale as $T^{1/2}$. Quantum theory replaces n by n_{imp}. Quantum statistics (Fermi velocity instead of thermal velocity) correctly gives the impurity scattering part to be independent of T

Conductivity: $\vec{j} = \overleftrightarrow{\sigma} \cdot \vec{E}$

Example – tetragonal or hexagonal symmetry.

$$\overleftrightarrow{\sigma} = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

j is in A/m^2, E in V/m, σ is in $(\Omega m)^{-1}$, ρ=1/σ is resistivity (μΩcm)

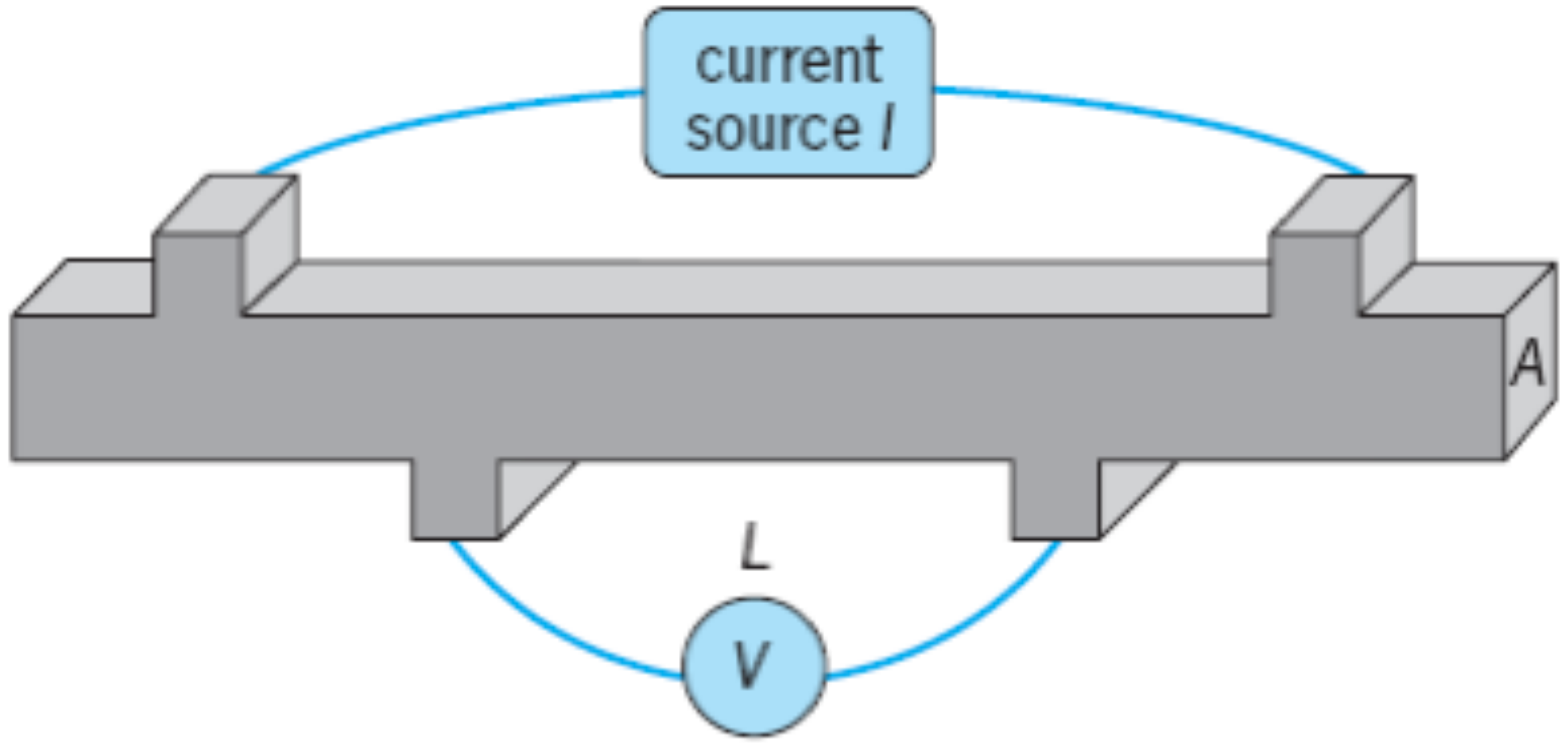


Fig. 1. Schematic of a four-probe measurement of electrical conductivity. Current *I* is fed through the outer leads, and voltage drop *V* is measured on the inner leads. In this way the contact potential drop experienced by the applied current is localized at the junctions with the outer leads. Measuring *V* with minimal current through the voltmeter minimizes the contact potential contribution to the measured conductance $G = I/V$.

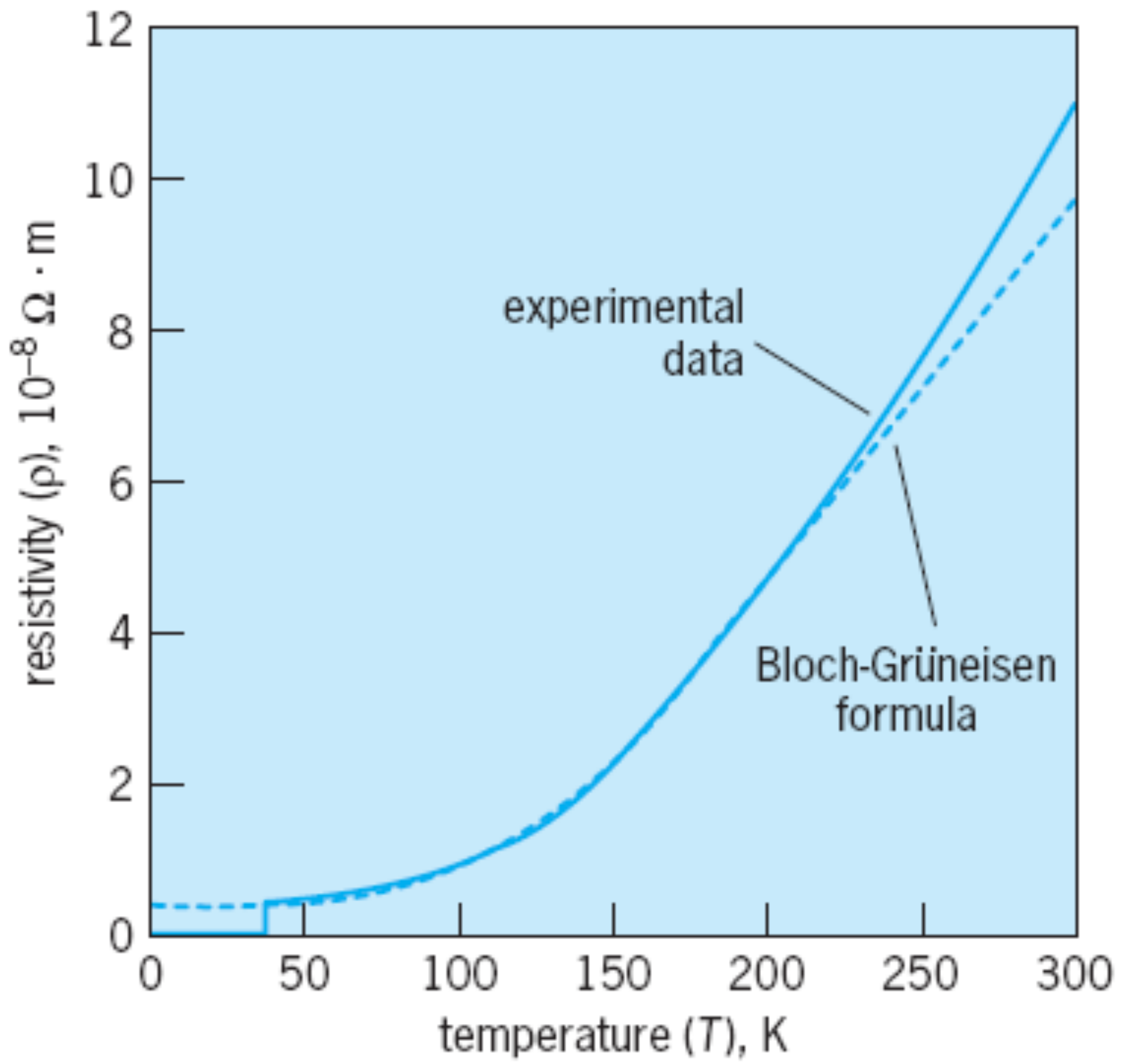
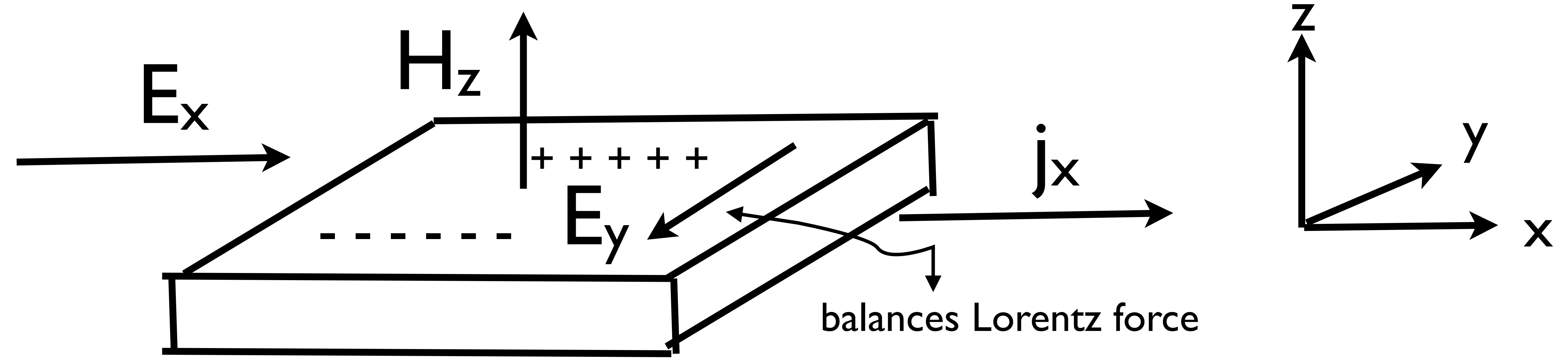


Fig. 2. Electrical conductivity of magnesium diboride (MgB_2), plotted, as is customary for metals, as resistivity (ρ) versus temperature (*T*). The unit $10^{-8}\ \Omega \cdot m$ is often written as 1 micro-ohm cm or 1 μΩ-cm. Below $T = 40$ K $= -233°C$, MgB_2 is a superconductor with $\rho = 0$. A fit using Bloch-Grünisen theory, explained in the text, is also shown. (*Data from Z. X. Ye et al., Electron scattering dependence of dendritic magnetic instability in superconducting* MgB_2 *films, Appl. Phys. Lett., 86:5284–5286, 2004*)

typical metallic conductivity: ρ ~ 10 μΩcm = 10^{-7} Ωm ➔ σ = 10^{7} $(\Omega m)^{-1}$

E = 1 V/m ➔ j = 10^{7} A/m^2



Hall Effect

$$\rho_{xx} = \frac{E_x}{j_x}$$

Tranverse magnetoresistance

$$\rho_{yx} = \frac{E_y}{j_x} = R_H H \Longrightarrow R_H = \frac{E_y}{j_x H}$$

$$Lorentz \quad \vec{f} = \frac{-e}{c} \vec{v} \times \vec{H}$$

- Hall E_y field⇒ negative y direction ⇒ R_h negative (reverses if there are positive carriers)
- Hall effect used to measure the nature of carriers (e^- or holes)

$$\vec{f} = -e(\vec{E} + \frac{\vec{v}}{c} \times \vec{H})$$

$$\frac{d\vec{p}}{dt} = -e(\vec{E} + \frac{\vec{p}}{mc} \times \vec{H}) - \frac{\vec{p}}{\tau} = 0 \Longleftarrow steady \quad state$$

$$\sigma_0 = \frac{ne^2 \tau}{m}$$

$$\vec{j} = -ne\vec{v} = -ne\vec{p}/m$$

- Solving in the x and y axis and balancing forces (j_y=0, no current along y)we arrive to:

$$E_x = \frac{j_x}{\sigma_0}$$

$$E_y = \frac{-\omega_c \tau}{\sigma_0} j_x = \frac{-\frac{eH}{mc}\tau}{\frac{ne^2\tau}{m}} j_x = -\frac{H}{nec} j_x$$

$$R_h = \frac{E_y}{H j_x} = \frac{-1}{nec} \Longleftarrow$$

Yields sign and density of charge carriers

Problems

- R_H can be -
- R_H depends on H
- R_H depends on T